3.1 Výpočet neurčitého integrálu

3.1.41 $\int \frac{1}{\left(1+e^x\right)^2}dx$

rozšíříme výrazem e^x a volíme substituci: $t=\varphi(x)=e^x \qquad I=R\xrightarrow{\varphi}J=(0,\infty)$

$\varphi'(x)=e^x \qquad J=(-\infty,1)\cup(-1,0)\cup(0,\infty)$

$$F(t)=\int \frac{1}{t(1+t)^2}dt$$

nalezneme rozklad na parciální zlomky: $\frac{1}{t(1+t)^2}=\frac{A}{t}+\frac{B}{1+t}+\frac{C}{(1+t)^2}$

t = -1 : C = -1
t = 0 : A = 1
t^2 : C = A + B => B = -1

$$F(t)=\int\frac{1}{t}dt-\int\frac{1}{1+t}dt-\int\frac{1}{(1+t)^2}dt=\ln t-\ln(1+t)+\frac{1}{1+t}+C$$

$$\underline{F(\varphi(x))=x-\ln(1+e^x)+\frac{1}{1+e^x}+C}$$

3.1.42 $\int \frac{1}{e^{2x}+e^x-2}dx$

rozšíříme výrazem e^x a volíme substituci: $t=\varphi(x)=e^x \qquad I=R\xrightarrow{\varphi}J=(0,\infty)$

$\varphi'(x)=e^x \qquad J=R-\{-2,0,1\}$

$$F(t)=\int \frac{1}{t(t^2+t-2)}dt$$

nalezneme rozklad na parciální zlomky: $\frac{1}{t(t^2+t-2)}=\frac{A}{t}+\frac{B}{t+2}+\frac{C}{t-1}$

t = 0 : $A=-\frac{1}{2}$

t = 1 : $C=\frac{1}{3}$

t =-2 : $B=\frac{1}{6}$

$$F(t)=-\frac{1}{2}\int\frac{1}{t}dt+\frac{1}{3}\int\frac{1}{t-1}dt+\frac{1}{6}\int\frac{1}{t+2}dt=-\frac{1}{2}\ln t+\frac{1}{3}\ln|t-1|+\frac{1}{6}\ln(t+2)+C$$

$$\underline{F(\varphi(x))=-\frac{1}{2}x+\frac{1}{3}\ln|e^x-1|+\frac{1}{6}\ln(e^x+2)+C}$$

3.1.43 $\int \frac{1}{1+e^{\frac{x}{2}}+e^{\frac{x}{3}}+e^{\frac{x}{6}}}dx$

upravíme a zavedeme substituci:

$$\int \frac{1}{1+\left(e^{\frac{x}{6}}\right)^3+\left(e^{\frac{x}{6}}\right)^2+e\frac{x}{6}}dx = 6\int \frac{\frac{1}{6}e^{\frac{x}{6}}}{e^{\frac{x}{6}}\left(1+\left(e^{\frac{x}{6}}\right)^3+\left(e^{\frac{x}{6}}\right)^2+e^{\frac{x}{6}}\right)}dx$$

$t=\varphi(x)=e^{\frac{x}{6}} \qquad I=R\xrightarrow{\varphi}J=(0,\infty)$

$\varphi'(x)=\frac{1}{6}e^{\frac{x}{6}} \qquad J=R-\{0,1\}$

$$F(t)=6\int \frac{1}{t(t^3+t^2+t+1)}dt$$

nalezneme rozklad na parciální zlomky; je zřejmé, že polynom t^3+t^2+t+1 má kořen $t=1$, tedy $t^3+t^2+t+1=(t+1)(t^2+1)$

$$\frac{1}{t(t^3+t^2+t+1)}=\frac{A}{t}+\frac{B}{t+1}+\frac{Ct+D}{t^2+1}$$

$t = -1$: $\quad B=-\frac{1}{2}$

$t = 0$: $\quad A=1$

t^3 : $\quad A+B+C=0 \Rightarrow C=-\frac{1}{2}$

t^1 : $\quad A+B+D=0 \Rightarrow D=-\frac{1}{2}$

$$F(t)=6\left[\int \frac{1}{t}dt-\frac{1}{2}\int \frac{1}{t+1}dt-\frac{1}{2}\int \frac{t+1}{t^2+1}dt\right]=6\ln t-3\ln(t+1)-\frac{3}{2}\ln(t^2+1)-3arctg\ t+C$$

$$F(\varphi(x))=x-3\ln\left(e^{\frac{x}{6}}+1\right)-\frac{3}{2}\ln\left(e^{\frac{x}{3}}+1\right)-3arctg\ e^{\frac{x}{6}}+C$$

3.1.44 $\int \frac{1+e^{\frac{x}{2}}}{\left(1+e^{\frac{x}{4}}\right)^2}dx$

upravíme a zavedeme substituci: $\int \frac{1+\left(e^{\frac{x}{4}}\right)^2}{\left(1+e^{\frac{x}{4}}\right)^2}dx = 4\int \frac{\frac{1}{4}e^{\frac{x}{4}}\left[1+\left(e^{\frac{x}{4}}\right)^2\right]}{e^{\frac{x}{4}}\left(1+e^{\frac{x}{4}}\right)^2}dx$

$t=\varphi(x)=e^{\frac{x}{4}} \qquad I=R \xrightarrow{\varphi} J=(0,\infty)$

$\varphi'(x)=\frac{1}{4}e^{\frac{x}{4}} \qquad J=R-\{-1,0\}$

$F(t)=4\int \frac{1+t^2}{t(1+t)^2}dt$

nalezneme rozklad na parciální zlomky: $\frac{1+t^2}{t(1+t)^2}=\frac{A}{t}+\frac{B}{1+t}+\frac{C}{(1+t)^2}$

t = -1 : C = -2
t = 0 : A = 1
t^2 : A + B = 1 => B = 0

$F(t)=4\int \frac{1}{t}dt-2\int \frac{1}{(1+t)^2}dt=4\ln t+8\frac{1}{1+t}+C$

$F(\varphi(x))=x+\frac{8}{1+e^{\frac{x}{4}}}+C$

3.1.45 $\int \frac{1}{\left(e^{x+1}+1\right)^2-\left(e^{x-1}+1\right)^2}dx$

upravíme a zavedeme substituci:

$\int \frac{1}{\left(e^{x+1}+1-e^{x-1}-1\right)\left(e^{x+1}+1+e^{x-1}+1\right)}dx=\int \frac{1}{e^x\left(e-\frac{1}{e}\right)\left[e^x\left(e+\frac{1}{e}\right)+2\right]}dx=$

$=\frac{1}{4\,sh\,1\,ch\,1}\int \frac{e^x}{e^{2x}\left(e^x+\frac{1}{ch\,1}\right)}dx$

$t=\varphi(x)=e^x \qquad I=R \xrightarrow{\varphi} J=(0,\infty)$

$\varphi'(x)=e^x \qquad J=R-\left\{-\frac{1}{ch\,1},\,0\right\}$

$$F(t)=\frac{1}{4\,sh\,1\,ch\,1}\int\frac{1}{t^2(t+\frac{1}{ch\,1})}dt$$

nalezneme rozklad na parciální zlomky: $\frac{1}{t^2\left(t+\frac{1}{ch\,1}\right)}=\frac{A}{t}+\frac{B}{t^2}+\frac{C}{t+\frac{1}{ch\,1}}$

$t=0$: B = ch 1

$t=-\frac{1}{ch\,1}$: C = ch^2 1

t^2 : A + C = 0 => A = - ch^2 1

$$F(t)=-\frac{cth\,1}{4}\int\frac{1}{t}dt+\frac{1}{4\,sh\,1}\int\frac{1}{t^2}dt+\frac{cth\,1}{4}\int\frac{1}{t+\frac{1}{ch\,1}}dt=$$

$$=-\frac{cth\,1}{4}\ln t-\frac{1}{4\,sh\,1}\frac{1}{t}+\frac{cth\,1}{4}\ln\left(t+\frac{1}{ch\,1}\right)+C_0$$

$$\underline{F(\varphi(x))=-\frac{cth\,1}{4}\left(x-\ln\left[(ch\,1)e^x+1\right]\right)-\frac{1}{4(sh\,1)e^x}+C}$$

3.1.46 Vypočítejte integrály z racionálních funkcí:

$$a)\int\frac{x^5-2x^2+5}{x^2-4x+4}dx=\int x^3+4x^2+12x+30+\frac{72x-115}{x^2-4x+4}dx=\frac{x^4}{4}+\frac{4}{3}x^3+6x^2+30x+$$

$$+\int\frac{72x-115}{x^2-4x+4}dx=\frac{x^4}{4}+\frac{4}{3}x^3+6x^2+30x+\int\frac{29}{(x-2)^2}dx+\int\frac{72}{x-2}=\underline{\frac{x^4}{4}+\frac{4}{3}x^3+6x^2+}$$

$$\underline{+30x-\frac{29}{x-2}+72\ln|x-2|+C}$$

$$b)\int\frac{dx}{x^4+x^2}=\int\left(\frac{1}{x^2}-\frac{1}{x^2+1}\right)dx=\underline{-\frac{1}{x}-arctg\,x+C}$$

$$c)\int\frac{x^2+2}{x^3-3x-2}dx=\int\frac{x^2+2}{(x+1)^2(x-2)}dx=\int\frac{dx}{3\cdot(x+1)}-\int\frac{dx}{(x+1)^2}+\frac{2}{3}\int\frac{dx}{(x-2)}=$$

$$=\underline{\frac{1}{3}\ln|x+1|+\frac{2}{3}\ln|x-2|+\frac{1}{x+1}+C}$$

$$d)\int \frac{x^2+2x+6}{x^2+2x+5}dx=\int dx+\int \frac{1}{x^2+2x+5}dx=x+\frac{1}{4}\int \frac{dx}{1+\left(\frac{x+1}{2}\right)^2}=\underline{x+\frac{1}{2}arctg\left(\frac{x+1}{2}\right)+C}$$

$$e)\int \frac{2x+3}{\left(x^2+2x+5\right)^3}dx=\int \frac{2x+2}{\left(x^2+2x+5\right)^3}dx+\int \frac{dx}{\left(x^2+2x+5\right)^3}=\left|\begin{matrix} t=x^2+2x+5 \\ dt=(2x+2)dx \end{matrix}\right|=\int \frac{dt}{t^3}+$$

$$+\int \frac{dx}{\left((x+1)^2+4\right)^3}=\left|\begin{matrix} x+1=z \\ dx=dz \end{matrix}\right|=-\frac{1}{2\,t^2}+\int \frac{dz}{\left(z^2+4\right)^3}=-\frac{1}{2\cdot\left(x^2+2x+5\right)^2}+\frac{1}{16}\frac{z}{\left(z^2+4\right)^2}+$$

$$+\frac{3}{16}\int \frac{dz}{\left(z^2+4\right)^2}=-\frac{1}{2\cdot\left(x^2+2x+5\right)^2}+\frac{1}{16}\frac{(x+1)}{\left((x+1)^2+4\right)^2}+\frac{3}{16}\cdot\left[\frac{1}{8}\cdot\frac{z}{\left(z^2+4\right)}+\frac{1}{8}\int \frac{dz}{\left(z^2+4\right)}\right]=$$

$$=-\frac{1}{2\cdot\left(x^2+2x+5\right)^2}+\frac{1}{16}\frac{(x+1)}{\left(x^2+2x+5\right)^2}+\frac{3}{16}\left[\frac{x+1}{8\left(x^2+2x+5\right)}+\frac{1}{16}arctg\frac{x+1}{2}\right]+C=$$

$$=\underline{\frac{x-7}{16\cdot\left(x^2+2x+5\right)^2}+\frac{3(x+1)}{128\cdot\left(x^2+2x+5\right)}+\frac{3}{256}arctg\left(\frac{x+1}{2}\right)+C}$$

3.1.47 Vhodnou substitucí zjednodušte integrály (bez dopočítání výsledku):

$$a)\int \frac{x^3}{\left(3x^2+2\right)^2}dx=\left|\begin{matrix} 3x^2+2=u \\ du=6x\,dx \end{matrix}\right|=\int \frac{x^3}{u^2}\cdot\frac{du}{6x}=\int \frac{u-2}{3}\cdot\frac{1}{u^2}\cdot\frac{du}{6}=\frac{1}{18}\int \frac{1}{u}du-\frac{1}{9}\int \frac{1}{u^2}du=$$

$$=\underline{\frac{1}{18}\ln\left|3x^2+2\right|+\frac{1}{9}\left(\frac{1}{3x^2+2}\right)+C}$$

$$b)\int \frac{x^3}{\left(x^8+1\right)^2}dx=\left|\begin{matrix} x^4=t \\ dt=4x^3\,dx \end{matrix}\right|=\frac{1}{4}\int \frac{dt}{\left(t^2+1\right)^2}=\ldots$$

$$c)\int \frac{x^5}{x^{12}-1}dx=\left|\begin{matrix} x^6=t \\ dt=6x^5dx \end{matrix}\right|=\frac{1}{6}\int \frac{dt}{t^2-1}=\ldots$$

3.1.48 Převeďte na integrál z racionální funkce (bez dopočítání výsledku):

$$a)\int \frac{dx}{1+\sqrt[3]{x+1}} = \left| \begin{array}{l} x+1=t^3 \\ 3t^2\,dt = dx \end{array} \right| = 3\int \frac{t^2}{1+t}\,dt = \ldots$$

$$b)\int \frac{\sqrt[3]{x}+1}{\sqrt{x}}\,dx = \left| \begin{array}{l} x=u^6 \\ dx = 6u^5\,du \end{array} \right| = 6\int \left(u^4+u^2\right)du = \ldots$$

$$c)\int \frac{dx}{\sqrt[3]{(x+1)^2\cdot(x-1)}} = \left| \begin{array}{l} \sqrt[3]{(x+1)^2\cdot(x-1)} = t\cdot(x+1) \\ (x+1)^2\cdot(x-1) = t^3\cdot(x+1)^3 \\ dx = \dfrac{6t^2}{\left(1-t^3\right)^2}\,dt \end{array} \right| = \int \frac{1}{t\cdot\left(\dfrac{t^3+1}{1-t^3}+1\right)}\cdot\frac{6t^2}{\left(1-t^3\right)^2}\,dt =$$

$$= 3\int \frac{t}{\left(1-t^3\right)^2}\,dt = \ldots$$

$$d)\int \frac{dx}{\sqrt{(x-1)^3\cdot(x-2)}} = \int \frac{dx}{(x-1)\sqrt{(x-1)\cdot(x-2)}} = \left| \begin{array}{l} \sqrt{x^2-3x+2} = t\cdot(x-1) \\ x^2-3x+2 = t^2\cdot(x-1)^2 \\ x = \dfrac{2-t^2}{1-t^2} \\ dx = \dfrac{-2t\left(1-t^2\right)+2t\left(2-t^2\right)}{\left(1-t^2\right)^2}\,dt \end{array} \right| =$$

$$= \int \frac{2t}{\left(1-t^2\right)t\left(\dfrac{2-t^2}{1-t^2}-1\right)^2}\,dt = 2\int dt = \ldots$$

3.1.49 Užitím Eulerovy substituce převeďte na integrál z racionálních funkcí (který již nemusíte dopočítávat):

$$a)\int \frac{dx}{\sqrt{4x^2+2x+1}} = \left| \begin{array}{l} \sqrt{4x^2+2x+1} = \sqrt{4}\,x+t \\ 4x^2+2x+1 = (2x+t)^2 \\ dx = \dfrac{t-t^2-1}{(1-2t)^2}\,dt \end{array} \right| = \int \frac{1}{\dfrac{t^2-1}{1-2t}+t}\cdot\frac{t-t^2-1}{(1-2t)^2}\,dt = \int \frac{1}{1-2t}\,dt = \ldots$$

$$b)\int \frac{x\,dx}{\sqrt{4+2x-x^2}} = \left| \begin{array}{l} \sqrt{4+2x-x^2} = xt+\sqrt{4} \\ 4+2x-x^2 = x^2t^2+4xt+4 \\ dx = 4\,\dfrac{t^2-t-1}{\left(t^2+1\right)^2}\,dt \end{array} \right| = \int \frac{-2\,\dfrac{2t-1}{t^2+1}}{-2\,\dfrac{2t-1}{t^2+1}\,t+2}\cdot 4\,\frac{t^2-t-1}{\left(t^2+1\right)^2}\,dt =$$

$$= 4\int \frac{2t-1}{\left(t^2+1\right)^2}\,dt = \ldots$$

3.1.50 Vypočtěte: $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx = \int \frac{1}{\sin^2 x} dx - \int \frac{1}{\cos^2 x} dx = \underline{-tgx - \cot gx + C}$$

3.1.51 Vypočtěte: $\int \frac{1+2x^2}{x^2(1+x^2)} dx$

$$\int \frac{1+2x^2}{x^2(1+x^2)} dx = \int \frac{1+x^2}{x^2(1+x^2)} dx + \int \frac{x^2}{x^2(1+x^2)} dx = \int \frac{1}{x^2} dx + \int \frac{1}{1+x^2} dx = \underline{-\frac{1}{x} + arctgy + C}$$

3.1.52 Vypočtěte: $\int \frac{x^4}{\sqrt{4+x^5}} dx$

$$\int \frac{x^4}{\sqrt{4+x^5}} dx = \frac{1}{5} \int \frac{5x^4}{4+x^5} dx = \underline{\frac{1}{5} \ln\left|4+x^5\right| + C}$$

3.1.53 Vypočtěte: $\int \frac{(\sqrt{x}+2)^3}{x} dx$

$$\int \frac{(\sqrt{x}+2)^3}{x} dx = \int \frac{\sqrt{x^3} + 6x + 12\sqrt{x} + 8}{x} dx = \int \left(x^{\frac{1}{2}} + 6 + 12x^{-\frac{1}{2}} + 8x^{-1} \right) dx =$$

$$= \underline{\frac{2}{3} x\sqrt{x} + 6x + 24\sqrt{x} + 8\ln|x| + C}$$

3.1.54 Vypočtěte: $\int \frac{2x+5}{\sqrt{9-9x^2}} dx$

$$\int \frac{2x+5}{\sqrt{9-9x^2}} dx = \int \frac{2x}{3\sqrt{1-x^2}} dx + \int \frac{5}{3\sqrt{1-x^2}} dx = \underline{-\frac{2}{3}\sqrt{1-x^2} + \frac{5}{3}\arcsin x + C}$$

3.1.55 Vypočtěte: $\int \sqrt[5]{(8-3x)^6} dx$

$$\int \sqrt[5]{(8-3x)^6} dx = \int (8-3x)^{\frac{5}{6}} dx = \underline{-\frac{5}{33}(8-3x)^{\frac{11}{6}} + C}$$

3.1.56 Vypočtěte užitím vhodné substituce: $\int \frac{x+1}{\sqrt[3]{3x+1}} dx$

$$\int \frac{x+1}{\sqrt[3]{3x+1}}dx = \left| \begin{matrix} 3x+1 = z^3 \rightarrow x = \dfrac{z^3-1}{3} \\ dx = z^2 dz \end{matrix} \right| = \int \frac{z^3+2}{3}\cdot\frac{z^2 dz}{z} = \frac{1}{3}\int (z^4+2z)dz = \frac{z^5}{5}+\frac{z^2}{3} =$$

$$= \underline{\frac{1}{15}\sqrt[3]{(3x+1)^5}+\frac{1}{3}\sqrt[3]{(3x+1)^2}+C}$$

3.1.57 Vypočtěte užitím vhodné substituce: $\int \frac{xdx}{\sqrt{2x+1}+1}$

$$\int \frac{xdx}{\sqrt{2x+1}+1} = \left| \begin{matrix} 2x+1 = r^2 \rightarrow x = \dfrac{r^2-1}{2} \\ dx = rdr \end{matrix} \right| = \int \frac{r^2-1}{2(r+1)}rdr = \frac{1}{2}\int (r^2-r)dr = \frac{r^3}{6}-\frac{r^2}{4}+C =$$

$$= \underline{\frac{\sqrt{(2x+1)^3}}{6}-\frac{2x+1}{4}+C}$$

3.1.58 Vypočtěte užitím vhodné substituce: $\int \frac{x^3dx}{\sqrt{x^2+2}}$

$$\int \frac{x^3dx}{\sqrt{x^2+2}} = \left| \begin{matrix} x^2+2 = t^2 \\ 2xdx = 2tdt \end{matrix} \right| = \int \frac{(t^2-2)}{t}tdt = \int (t^2-2)dt = \frac{t^3}{3}-2t+C =$$

$$= \underline{\frac{1}{3}\sqrt{(x^2+2)^3}-2\sqrt{x^2+2}+C}$$

3.1.59 Vypočtěte užitím vhodné substituce: $\int \frac{dx}{e^x-1}$

$$\int \frac{dx}{e^x-1} = \int \frac{1-e^x+e^x}{e^x-1}dx = \int \frac{e^x}{e^x-1}dx - \int 1dx = \left| \begin{matrix} e^x-1 = t \\ e^x dx = dt \end{matrix} \right| = \int \frac{dt}{t} - \int 1dx = \underline{\ln\left|e^x-1\right| - x + C}$$

3.1.60 Vypočtěte užitím vhodné substituce: $\int e^{\sqrt{x}}dx$

$$\int e^{\sqrt{x}}dx = \left| \begin{matrix} x = z^2 \\ dx = 2zdz \end{matrix} \right| = 2\int e^z zdz = \left| \begin{matrix} u = z \quad u' = 1 \\ v' = e^z \quad v = e^z \end{matrix} \right| = 2ze^z - 2\int e^z dz = 2ze^z - 2e^z + C =$$

$$= \underline{2e^{\sqrt{x}}(\sqrt{x-1})+C}$$

3.1.61 Vypočtěte užitím vhodné substituce: $\int \frac{x}{\sqrt{x^2-a^2}}dx$

$$\int \frac{x}{\sqrt{x^2-a^2}}dx = \left| \begin{matrix} x^2-a^2 = z^2 \\ 2xdx = 2zdz \end{matrix} \right| = \int \frac{zdz}{z} = z + C = \underline{\sqrt{x^2+a^2}+C}$$

3.1.62 Užitím metody per-partes vypočtěte: $\int x^2 \cos 2x dx$

$$\int x^2 \cos 2x dx = \begin{vmatrix} u = x^2 & u' = 2x \\ v' = \cos 2x & v = \frac{1}{2}\sin 2x \end{vmatrix} = \frac{1}{2}x^2 \sin 2x - \int x \sin 2x dx =$$

$$= \begin{vmatrix} u = x & u' = 1 \\ v' = \sin 2x & v = -\frac{1}{2}\cos 2x \end{vmatrix} = \frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \int \frac{1}{2}\cos 2x dx =$$

$$= \underline{\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x + C}$$

3.1.63 Užitím metody per-partes vypočtěte: $\int x arctgx dx$

$$\int x arctgx dx = \begin{vmatrix} u = arctgx & u' = \frac{1}{1+x^2} \\ v' = x & v = \frac{1}{2}x^2 \end{vmatrix} = \frac{1}{2}x^2 arctgx - \frac{1}{2}\int \frac{x^2}{1+x^2} dx =$$

$$= \frac{1}{2}x^2 arctgx - \frac{1}{2}\int 1 dx + \frac{1}{2}\int \frac{1}{1+x^2} dx = \underline{\frac{1}{2}x^2 arctgx - \frac{1}{2}x} + \frac{1}{2} arctgx + C$$

3.1.64 Užitím metody per-partes vypočtěte: $\int \ln^2 x dx$

$$\int \ln^2 x dx = \begin{vmatrix} u = \ln^2 x & u' = 2\ln x \cdot \frac{1}{x} \\ v' = 1 & v = x \end{vmatrix} = x \cdot \ln^2 x - \int 2\ln x dx = \begin{vmatrix} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{vmatrix} =$$

$$= x \cdot \ln^2 x - 2(x \ln x - \int \frac{1}{x} x dx) = \underline{x \cdot \ln^2 x - 2x \ln x + 2x + C}$$

3.1.65 Užitím rekurentních vzorců pro I_c , I_S , I_n vypočtěte: $\int e^{3x} \cos 4x dx$

$$\int e^{3x} \cos 4x dx = \frac{4e^{3x} \sin 4x + 3e^{3x} \cos 4x}{25} + C = \underline{\frac{e^{3x}}{25}(4 \sin 4x + 3 \cos 4x) + C}$$

3.1.66 Užitím rekurentních vzorců pro I_c , I_S , I_n vypočtěte: $\int e^{4x} \sin 3x dx$

$$\int e^{4x} \sin 3x dx = \frac{4e^{4x} \sin 3x - 3e^{4x} \cos 3x}{25} + C = \underline{\frac{e^{4x}}{25}(4 \sin 3x - 3 \cos 3x) + C}$$

3.1.67 Užitím rekurentních vzorců pro I_c , I_S , I_n vypočtěte: $\int \frac{dx}{(x^2+1)^3}$

$$\int \frac{dx}{(x^2+1)^3} = \frac{x}{4}\cdot\frac{1}{(1+x^2)^2} + \frac{3}{4}\int \frac{1}{(x^2+1)^2}dx = \frac{x}{4(1+x^2)^2} + \frac{3}{4}\left(\frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2}dx\right) =$$

$$= \underline{\frac{x}{4(1+x^2)^2} + \frac{3x}{8(1+x^2)} + \frac{3}{8}arctgx + C}$$

3.1.68 Užitím rekurentních vzorců pro I_c , I_S , I_n vypočtěte: $\int \frac{dx}{(x^2+9)^2}$

$$\int \frac{dx}{(x^2+9)^2} = \frac{x}{18(x^2+9)} + \frac{1}{18}\int \frac{dx}{x^2+9} = \frac{x}{18(x^2+9)^2} + \frac{1}{162}\int \frac{1}{\left(\frac{x}{3}\right)^2+1}dx =$$

$$= \underline{\frac{x}{18(x^2+9)^2} + \frac{1}{54}arctg\frac{x}{3} + C}$$

3.1.69 Vypočtěte: $\int \frac{\cos 2x}{\cos^2 x \sin^2 x}dx$

$$\int \frac{\cos 2x}{\cos^2 x\sin^2 x}dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x}dx = \int \frac{1}{\sin^2 x}dx - \int \frac{1}{\cos^2 x}dx = \underline{-tgx - \cot gx + C}$$

3.1.70 Vypočtěte: $\int \frac{1+2x^2}{x^2(1+x^2)}dx$

$$\int \frac{1+2x^2}{x^2(1+x^2)}dx = \int \frac{1+x^2}{x^2(1+x^2)}dx + \int \frac{x^2}{x^2(1+x^2)}dx = \int \frac{1}{x^2}dx + \int \frac{1}{1+x^2}dx = \underline{-\frac{1}{x} + arctgy + C}$$

3.1.71 Vypočtěte: $\int \frac{x^4}{\sqrt{4+x^5}}dx$

$$\int \frac{x^4}{\sqrt{4+x^5}}dx = \frac{1}{5}\int \frac{5x^4}{4+x^5}dx = \underline{\frac{1}{5}\ln\left|4+x^5\right| + C}$$

3.1.72 Vypočtěte: $\int \frac{(\sqrt{x}+2)^3}{x}dx$

$$\int \frac{(\sqrt{x}+2)^3}{x}dx = \int \frac{\sqrt{x^3}+6x+12\sqrt{x}+8}{x}dx = \int\left(x^{\frac{1}{2}} + 6 + 12x^{-\frac{1}{2}} + 8x^{-1}\right)dx =$$

$$= \underline{\frac{2}{3}x\sqrt{x} + 6x + 24\sqrt{x} + 8\ln|x| + C}$$

3.1.73 Vypočtěte: $\int \frac{2x+5}{\sqrt{9-9x^2}}dx$

$$\int \frac{2x+5}{\sqrt{9-9x^2}}dx = \int \frac{2x}{3\sqrt{1-x^2}}dx + \int \frac{5}{3\sqrt{1-x^2}}dx = \underline{-\frac{2}{3}\sqrt{1-x^2} + \frac{5}{3}\arcsin x + C}$$

3.1.74 Vypočtěte: $\int \sqrt[5]{(8-3x)^6}\,dx$

$$\int \sqrt[5]{(8-3x)^6}\,dx = \int (8-3x)^{\frac{5}{6}}dx = \underline{-\frac{5}{33}(8-3x)^{\frac{11}{6}} + C}$$

3.1.75 Vypočtěte užitím vhodné substituce: $\int \frac{x+1}{\sqrt[3]{3x+1}}dx$

$$\int \frac{x+1}{\sqrt[3]{3x+1}}dx = \left| \begin{array}{l} 3x+1 = z^3 \rightarrow x = \dfrac{z^3-1}{3} \\ dx = z^2dz \end{array} \right| = \int \frac{z^3+2}{3}\cdot\frac{z^2dz}{z} = \frac{1}{3}\int (z^4+2z)dz = \frac{z^5}{5} + \frac{z^2}{3} =$$

$$= \underline{\frac{1}{15}\sqrt[3]{(3x+1)^5} + \frac{1}{3}\sqrt[3]{(3x+1)^2} + C}$$

3.1.76 Vypočtěte užitím vhodné substituce: $\int \frac{xdx}{\sqrt{2x+1}+1}$

$$\int \frac{xdx}{\sqrt{2x+1}+1} = \left| \begin{array}{l} 2x+1 = r^2 \rightarrow x = \dfrac{r^2-1}{2} \\ dx = rdr \end{array} \right| = \int \frac{r^2-1}{2(r+1)}rdr = \frac{1}{2}\int (r^2-r)dr = \frac{r^3}{6} - \frac{r^2}{4} + C =$$

$$= \underline{\frac{\sqrt{(2x+1)^3}}{6} - \frac{2x+1}{4} + C}$$

3.1.77 Vypočtěte užitím vhodné substituce: $\int \frac{x^3dx}{\sqrt{x^2+2}}$

$$\int \frac{x^3dx}{\sqrt{x^2+2}} = \left| \begin{array}{l} x^2+2 = t^2 \\ 2xdx = 2tdt \end{array} \right| = \int \frac{(t^2-2)}{t}tdt = \int (t^2-2)dt = \frac{t^3}{3} - 2t + C =$$

$$= \underline{\frac{1}{3}\sqrt{(x^2+2)^3} - 2\sqrt{x^2+2} + C}$$

3.1.78 Vypočtěte užitím vhodné substituce: $\int \frac{dx}{e^x - 1}$

$$\int \frac{dx}{e^x-1} = \int \frac{1-e^x+e^x}{e^x-1}dx = \int \frac{e^x}{e^x-1}dx - \int 1dx = \begin{vmatrix} e^x - 1 = t \\ e^x dx = dt \end{vmatrix} = \int \frac{dt}{t} - \int 1dx = \underline{\ln\left|e^x - 1\right| - x + C}$$

3.1.79 Vypočtěte užitím vhodné substituce: $\int e^{\sqrt{x}} dx$

$$\int e^{\sqrt{x}} dx = \begin{vmatrix} x = z^2 \\ dx = 2zdz \end{vmatrix} = 2\int e^z zdz = \begin{vmatrix} u = z \ u' = 1 \\ v' = e^z \ v = e^z \end{vmatrix} = 2ze^z - 2\int e^z dz = 2ze^z - 2e^z + C =$$

$$= \underline{2e^{\sqrt{x}}(\sqrt{x-1}) + C}$$

3.1.80 Vypočtěte užitím vhodné substituce: $\int \frac{x}{\sqrt{x^2 - a^2}} dx$

$$\int \frac{x}{\sqrt{x^2-a^2}} dx = \begin{vmatrix} x^2 - a^2 = z^2 \\ 2xdx = 2zdz \end{vmatrix} = \int \frac{zdz}{z} = z + C = \underline{\sqrt{x^2 + a^2} + C}$$

3.1.81 Užitím metody per-partes vypočtěte: $\int x^2 \cos 2xdx$

$$\int x^2 \cos 2xdx = \begin{vmatrix} u = x^2 & u' = 2x \\ v' = \cos 2x & v = \frac{1}{2}\sin 2x \end{vmatrix} = \frac{1}{2}x^2 \sin 2x - \int x \sin 2xdx =$$

$$= \begin{vmatrix} u = x & u' = 1 \\ v' = \sin 2x & v = -\frac{1}{2}\cos 2x \end{vmatrix} = \frac{1}{2}x^2 \sin 2x + \frac{1}{2}x\cos 2x - \int \frac{1}{2}\cos 2xdx =$$

$$= \underline{\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x\cos 2x - \frac{1}{4}\sin 2x + C}$$

3.1.82 Užitím metody per-partes vypočtěte: $\int xarctgxdx$

$$\int xarctgxdx = \begin{vmatrix} u = arctgx & u' = \frac{1}{1+x^2} \\ v' = x & v = \frac{1}{2}x^2 \end{vmatrix} = \frac{1}{2}x^2 arctgx - \frac{1}{2}\int \frac{x^2}{1+x^2}dx =$$

$$= \frac{1}{2}x^2 arctgx - \frac{1}{2}\int 1dx + \frac{1}{2}\int \frac{1}{1+x^2}dx = \underline{\frac{1}{2}x^2 arctgx - \frac{1}{2}x} + \frac{1}{2}arctgx + C$$

3.1.83 Užitím metody per-partes vypočtěte: $\int \ln^2 xdx$

$$\int \ln^2 x dx = \begin{vmatrix} u = \ln^2 x & u' = 2\ln x \cdot \frac{1}{x} \\ v' = 1 & v = x \end{vmatrix} = x \cdot \ln^2 x - \int 2\ln x dx = \begin{vmatrix} u = \ln x & u' = \frac{1}{x} \\ v' = 1 & v = x \end{vmatrix} =$$

$$= x \cdot \ln^2 x - 2(x \ln x - \int \frac{1}{x} x dx) = \underline{x \cdot \ln^2 x - 2x \ln x + 2x + C}$$

3.1.84 Užitím rekurentních vzorců pro I_c , I_S , I_n vypočtěte: $\int e^{3x} \cos 4x dx$

$$\int e^{3x} \cos 4x dx = \frac{4e^{3x} \sin 4x + 3e^{3x} \cos 4x}{25} + C = \underline{\frac{e^{3x}}{25}(4\sin 4x + 3\cos 4x) + C}$$

3.1.85 Užitím rekurentních vzorců pro I_c , I_S , I_n vypočtěte: $\int e^{4x} \sin 3x dx$

$$\int e^{4x} \sin 3x dx = \frac{4e^{4x} \sin 3x - 3e^{4x} \cos 3x}{25} + C = \underline{\frac{e^{4x}}{25}(4\sin 3x - 3\cos 3x) + C}$$

3.1.86 Užitím rekurentních vzorců pro I_c , I_S , I_n vypočtěte: $\int \frac{dx}{(x^2+1)^3}$

$$\int \frac{dx}{(x^2+1)^3} = \frac{x}{4} \cdot \frac{1}{(1+x^2)^2} + \frac{3}{4} \int \frac{1}{(x^2+1)^2} dx = \frac{x}{4(1+x^2)^2} + \frac{3}{4}\left(\frac{x}{2(1+x^2)} + \frac{1}{2} \int \frac{1}{1+x^2} dx \right) =$$

$$= \underline{\frac{x}{4(1+x^2)^2} + \frac{3x}{8(1+x^2)} + \frac{3}{8} arctg x + C}$$

3.1.87 Užitím rekurentních vzorců pro I_c , I_S , I_n vypočtěte: $\int \frac{dx}{(x^2+9)^2}$

$$\int \frac{dx}{(x^2+9)^2} = \frac{x}{18(x^2+9)} + \frac{1}{18} \int \frac{dx}{x^2+9} = \frac{x}{18(x^2+9)^2} + \frac{1}{162} \int \frac{1}{\left(\frac{x}{3}\right)^2 + 1} dx =$$

$$= \underline{\frac{x}{18(x^2+9)^2} + \frac{1}{54} arctg \frac{x}{3} + C}$$