<u>3.2 Výpočet určitého integrálů</u>

3.2.1 Pomocí Eulerových vzorců vypočtěte $\int e^{2x} \cos x \, dx$.

$$\int e^{2x} \cos x \, dx = \underline{e^{2x} \cdot \frac{\sin x + 2\cos x}{5} + C}$$

3.2.2 Vypočtěte užitím Newtonova vzorce s tím, že nejprve najdete potřebné primitivní funkce:

<u>Řešení</u>:

$a) \int_{-\pi}^{\pi} \cos^5 x \, dx$

$$\int \cos^5 x \, dx = \int \cos x \cdot \left(\cos^2 x\right)^2 dx = \int \cos x \cdot \left(1 - \sin^2 x\right)^2 dx = \left| \begin{array}{l} \sin x = t \\ \cos x \, dx = dt \end{array} \right| = \int \left(1 - t^2\right)^2 dt =$$

$$= t - \frac{2}{3} t^3 + \frac{1}{5} t^5 + C = \sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + C$$

$$\int_{-\pi}^{\pi} \cos^5 x \, dx = \left[\sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x \right]_{-\pi}^{\pi} = \underline{0}$$

$b) \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\sin x}$

$$\int \frac{dx}{\sin x} = \int \frac{1 + t^2}{2t} \cdot \frac{2}{1 + t^2} dt = \ln|t| + C = \ln\left| tg \frac{x}{2} \right| + C$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\sin x} = \left[\ln\left| tg \frac{x}{2} \right| \right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \underline{\ln \frac{tg \frac{\pi}{6}}{tg \frac{\pi}{12}}}$$

$c) \int_{0}^{\frac{\pi}{2}} \cos 2x \sin 4x \, dx$

$$\int \cos 2x \sin 4x \, dx = \frac{1}{2} \int \left(\sin 2x + \sin 6x\right) dx$$

$$\int_{0}^{\frac{\pi}{2}} \cos 2x \sin 4x \, dx = \left[-\frac{\cos^2 x}{4} - \frac{\cos 6x}{12} \right]_{0}^{\frac{\pi}{2}} = \underline{\frac{2}{3}}$$

$$d)\ \int_0^{2\pi} \cos x \cos 3x \cos 5x\ dx$$

$$\int \cos x \cos 3x \cos 5x\ dx = \frac{1}{2}\int [\cos 2x + \cos 8x]\cdot \cos x\ dx = \frac{1}{4}\int (\cos x + \cos 3x + \cos 7x +$$

$$+ \cos 9x)dx = \frac{1}{4}\sin x + \frac{1}{12}\sin 3x + \frac{1}{28}\sin 7x + \frac{1}{36}\sin 9x + C$$

$$\int_0^{2\pi} \cos x \cos 3x \cos 5x\ dx = \left[\frac{1}{4}\sin x + \frac{1}{12}\sin 3x + \frac{1}{28}\sin 7x + \frac{1}{36}\sin 9x\right]_0^{2\pi} = \underline{0}$$

3.2.3. Vypočtěte $\int_0^{\frac{\pi}{4}} tg^3\ x\ dx$.

Řešení:

$$\int tg^3\ x\ dx = \int \frac{\sin x(1-\cos^2 x)}{\cos^3 x}dx = \left| \begin{matrix} \cos x = t \\ \sin x\, dx = -\,dt \end{matrix} \right| = -\int \frac{1-t^2}{t^3}dt = \frac{1}{2t^2} + \ln|t| + C =$$

$$= \frac{1}{2\cos^2 x} + \ln|\cos x| + C$$

$$\int_0^{\frac{\pi}{4}} tg^3\ x\ dx = \left[\frac{1}{2\cos^2 x} + \ln|\cos x|\right]_0^{\frac{\pi}{4}} = 1 + \ln\frac{\sqrt{2}}{2} - \frac{1}{2} - \ln 1 = \underline{\frac{1}{2} + \ln\frac{\sqrt{2}}{2}}$$

3.2.4 Příklady na 1 řádek:

$$a)\ \int_{\frac{\pi}{2}}^{\pi} \sqrt{1+\cos x}\ dx = \sqrt{2}\int_{\frac{\pi}{2}}^{\pi} \left|\cos\frac{x}{2}\right| dx = 2\sqrt{2}\left[\left|\sin\frac{x}{2}\right|\right]_{\frac{\pi}{2}}^{\pi} = \underline{2(\sqrt{2}-1)}$$

$$b)\ \int_0^{\frac{\pi}{4}} \frac{\sin^3 x + 1}{\cos^2 x}dx = \int_0^{\frac{\pi}{4}} \frac{\sin^3 x}{\cos^2 x}dx + \int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x}dx = \left[\frac{1}{\cos x} + \cos x + tgx\right]_0^{\frac{\pi}{4}} = \underline{\frac{3}{2}\sqrt{2} - 1}$$

$$c)\ \int \frac{(\sin x - \cos x)^2}{\sin 2x}dx = \int \frac{\sin^2 x - 2\cos x \sin x + \cos^2 x}{2\cos x \sin x}dx = \frac{1}{2}\int \frac{\sin^2 x}{\cos x \sin x}dx - \int dx +$$

$$+ \frac{1}{2}\int \frac{\cos^2 x}{\cos x \sin x}dx = \left| \begin{matrix} t = \cos x \\ dt = -\sin x dx \end{matrix} \right| = -\frac{1}{2}\int \frac{1}{t}dt - x - \frac{1}{2}\int \frac{t}{1-t^2}dt =$$

$$= \underline{-\frac{1}{2}\ln|\cos x| - x + \frac{1}{2}\ln|\sin x| + K}$$

$$d)\ \int \sin\left(x + \frac{\pi}{6}\right)dx = \int \sin t\, dt = \underline{-\cos\left(x + \frac{\pi}{6}\right) + K}$$

3.2.5 Vypočtěte $\int_{-2}^{5}\left(|x+1|-3|x-3|\right)dx$.

$$\int_{-2}^{-1}(-x-1)dx-3\int_{-2}^{-1}(-x+3)\,dx=\left[-\frac{x^2}{2}-x\right]_{-2}^{-1}-3\left[-\frac{x^2}{2}+3x\right]_{-2}^{-1}=-13$$

$$\int_{-1}^{3}(x+1+3x-9)dx=\int_{-1}^{3}(4x-8)dx=\left[2x^2-8x\right]_{-1}^{3}=-16$$

$$\int_{3}^{5}(x+1-3x+9)dx=\int_{3}^{5}(-2x+10)dx=\left[-x^2+10x\right]_{3}^{5}=4$$

$$\int_{-2}^{5}\left(|x+1|-3|x-3|\right)dx=-13-16+4=\underline{-25}$$